BULGARIAN ACADEMY OF SCIENCES

CYBERNETICS AND INFORMATION TECHNOLOGIES · Volume **5**, No 1

Sofia . 2005

Software Decisions for a Neutral Portfolio of Securities*

Ivan Popchev, Nadya Velinova

Institute of Information Technologies, 1113 Sofia

Abstract: *The aim of this article is to demonstrate how some software programs and products can be used to solve problems, connected with the construction of a neutral portfolio of securities. Some practical examples for analyzing the neutral portfolio of securities are presented. The primary motivation for constructing a neutral portfolio of securities is to create an exposure to specific risk without mixing any directional risk.*

The software programs used for solving the financial problems have many advantages. These programs are highly portable and integratable Internet based portfolio management systems. They are helping major corporations, government entities and smaller companies to realize their goals.

Keywords: *risk neutrality, Neutral Portfolio of Securities, the "Greeks", sensitivities, software programs and calculators.*

I. Characteristics of the basic model

I. 1. Basic functions

It is well known that in 1973 Fischer Black and Myron Scholes developed the Black/ Scholes' model to evaluate European call options. The Black-Scholes models assumes that the options can be exercised only at expiration. It requires that both the risk free-rate and volatility of the underlying stock price remain constant over the period of analysis. The model also assumes that the underlying stock does not pay the dividends; adjustments can be made to correct such distributions.[5, 7]

The **Black-Scholes formula** is a mathematical formula for the theoretical value of the European put and call stock options that may be derived from the assumptions of the model:

* This research is supported by the National Science Fund of the Bulgarian Ministry of Education and Science, Contract No И1305/2003.

$$C = S[N(d_1)] - Xe^{-rt}[N(d_2)],$$

$$d_1 = \frac{\ln(S/X) + [r + (\sigma^2/2)]t}{\sigma\sqrt{t}},$$

$$d_2 = d_1 - \sigma\sqrt{t},$$

$$P = C + Xe^{-rt} - S.$$

The variables are: C is a current value of the call option; P – a current value of the put option; S – a stock price; X – a strike price; t – time remaining until expiration; r – a current continuously compounded risk-free interest rate; σ – annual volatility of the stock price; $N(x)$ – standard normal cumulative distribution function.

In mathematical finance, the "**Greeks"** (delta, gamma, theta, vega, rho) are the quantities representing the market sensitivities of options or other derivatives, each measuring a different aspect of the risk in an option position and corresponding to the set of parameters on which the value of an instrument or portfolio of financial instruments is dependent. The name is used because most of the parameters are denoted by Greek letters [8, 11]

For computing the sensitivities to option analysis, the following mathematical formulas are used:

For the coefficient delta:

$$\delta_c = N(d_1),$$

$$\delta_p = N(d_1) - 1;$$

For the coefficient gamma:

$$\gamma_c = \frac{N'(d_1)}{S\sigma\sqrt{t}},$$

$$\gamma_p = \frac{N'(d_1)}{S\sigma\sqrt{t}},$$

$$N'(d_1) = \frac{e^{-\frac{d_1^2}{2}}}{\sqrt{2\pi}};$$

For the coefficient vega:

$$v_c = SN'(d_1)\sqrt{t},$$

$$v_p = SN'(d_1)\sqrt{t};$$

For the coefficient theta:

$$\theta_c = -\frac{SN'(d_1)\sigma}{2\sqrt{t}} - rXe^{-rt}N(d_2),$$

$$\theta_p = -\frac{SN'(d_1)\sigma}{2\sqrt{t}} + rXe^{-rt}N(-d_2),$$

For the coefficient rho:

$$\rho_c = Xte^{-rt}N(d_2),$$
$$\rho_p = -Xte^{-rt}N(-d_2).$$

The “**Greeks”** may be calculated using the different software calculators. In practice some calculators such as **Option Pricing Calculator by Peter Hoadley, Black-Scholes calculator**, etc. are used [15, 21, 22, 24, 32].

It is known that the term, **‘coefficient of sensitivity‘** is given a broad interpretation to cover components of variance so that the sensitivity coefficient shows the relationship of the individual uncertainty component to the standard deviation of the reported value for a test item. The coefficient of sensitivity relates to the result that is being reported and not to the method for estimating components of uncertainty. The coefficient of sensitivity of a given root variable is the partial derivative of the parameter value equation with respect to the root variable. The measure of the sensitivity of the call option price is the derivative of the call price with respect to time. This derivative is the precise measure of the instantaneous rate of change of the call option value that is due to passing of time. It is customary to assign Greek names to the sensitivity measures of options [8, 23]

I. 2. Practical examples

The practical examples for using the software calculators are presented below. The **Option Pricing Calculator by Peter Hoadley** is used to compute the coefficient of sensitivity and it will analyze the results obtained, depending on the change of given input parameters.

Example No 1. This example analyzes the **call options** by the stock price with measuring the price of the option. The input data are the following:

Input data	
1. Stock price	$ 81
2. Strike price	$ 80
3. Time to Expiration	60 days
4.Volatility	30 %
5. Risk-free rate	6%

After the use of the calculator by **Peter Hoadley** the following data are received:

Call Option						
Option price	Delta	Gamma	Theta	Vega	Rho	Position
$ 4.842	0.596	0.039	–0.039	0.127	0.072	in the money

The sensitivity chart is show in Appendix No 1.

Example No 2. This example analyzes the **put option** by the stock price measuring the coefficient Delta. The input data are the following:

Input data	
1. Stock price	$ 131
2. Strike price	$ 120
3. Time to Expiration	90 days
4.Volatility	25 %
5.Risk-free rate	5%

After using the calculator by **Peter Hoadley** the following data are available:

Put Option						
Option price	Delta	Gamma	Theta	Vega	Rho	Position
$ 1.838	–0.193	0.017	–0.021	0.180	–0.066	out the money

The sensitivity chart is show in Appendix No 2.

Example No 3. This example analyzes the **put option** by the volatility measuring the coefficient Gamma. The input data are the following:

Input data	
1. Stock price	$ 93
2. Strike price	$ 90
3. Time to Expiration	60 days
4.Volatility	20 %
5.Risk-free rate	7%

After the use of the calculator by **Peter Hoadley** the following data are obtained:

Put Option						
Option price	Delta	Gamma	Theta	Vega	Rho	Position
$ 1.356	–0.279	0.045	–0.016	0.128	–0.044	out the money

The sensitivity chart is shown in Appendix No 3.

Example No 4. This example analyzes the **call option** by the days to expiration with a measure of the coefficient Theta. The input data are thefollowing:

Input data	
1. Stock price	$ 73
2. Strike price	$ 60
3. Time to Expiration	60 days
4.Volatility	30 %
5.Rsk-free rate	6 %

After use of the calculator by **Peter Hoadley** the following data are obtained:

Call option						
Option price	Delta	Gamma	Theta	Vega	Rho	Position
$ 13.738	0.960	0.010	–0.016	0.027	0.093	in the money

The sensitivity chart is shown in Appendix No 4.

Example No 5. This example analyzes the **call option** by the strike price measuring the coefficient Rho. The input data are the following:

Input data	
1. Stock price	$ 84
2. Strike price	$ 80
3. Time to Expiration	90 days
4.Volatility	15 %
5.Risk-free rate	6 %

After the use of the calculator by **Peter Hoadley** the following data are obtained:

Call option						
Option price	Delta	Gamma	Theta	Vega	Rho	Position
$ 5.837	0.814	0.043	–0.020	0.114	0.155	in the money

The sensitivity chart is shown in Appendix No 5.

On the basis of the obtained results from the reviewed examples, the following conclusions can be drawn:

• The **Delta** coefficient has a max value with a lower value of the underlying asset and of course, the highest option price, and a min value – for an option with the lowest price.

• The **Gamma** coefficient has a max value with an average value of the underlying asset and of course, the lowest price of the option, and a min value – for an option with the highest price.

• Coefficient **Theta** has a max value with the lowest value of the underlying asset and of course the highest option price, and a min value – for an option with an average price.

• Coefficient **Vega** has a max value with the highest value of the underlying asset and with a comparatively low price of the option, and a min value – for an option with the highest price and a high value of the standard deviation.

• Coefficient **Rho** has a max value with a lower value of the underlying asset and with a comparatively high price of the option and a min value – for an option with the highest price of the base share and the lowest value of the risk-free interest rate.

These conclusions are corresponding to the conclusions in [8, 11].

II. Construction and analysis of the neutral portfolio of securities

II. 1. Construction of the delta neutral portfolio

Delta-neutral positions are very often used by options traders to create or offset exposure to option risks without being subject to directional market risk.

The term "delta-neutral" refers to any strategy where the sum of the deltas of the positions is equal to zero. Being delta-neutral means your portfolio consists of positions with positive and negative deltas that balance out, or that bring the net change of the position to zero. In other words, the response to market movements is neutralized. Given that the delta changes with fluctuating underlying prices, this neutralization is only valid for a certain narrow price range. Depending on whether a delta-neutral position is based on a long or on a short option leg, market volatility may be beneficial (long options) or harmful (short options) [1, 9, 18, 29, 30]

For the example of the theoretical model a delta-neutral portfolio is set in two given securities, choosing n_1 and n_2 so that $n_1\Delta_1 + n_2\Delta_2 = 0$. Solving this, we must set $n_1/n_2 = -\Delta_2/\Delta_1$. Such portfolios are useful for option market makers who must take positions in options but do not want to risk losses because of unfavorable asset price changes. They are also useful for investors who believe that options can be identified which are mispriced relatively to each other, but do not have any opinion about the direction of changes in the underlying asset price.

Delta-Neutral Portfolios: $n_1\Delta_1 + n_2\Delta_2 = 0 \Rightarrow n_1/n_2 = -\Delta_2/\Delta_1$

Case 1: Neutral Hedge

- buy 1 share ($\Delta_1 = 1$)
- sell 2 call options ($\Delta_2 = 0.5$)

$n_1/n_2 = -\Delta_2/\Delta_1 = -0.5/1 = -0.5.$

Case 2: Neutral Bullish Time Spread

- buy 6 call options ($\Delta_1 = 0.75$)
- sell 5 call options ($\Delta_2 = 0.91$)

$n_1/n_2 = -\Delta_2/\Delta_1 = -0.91/0.75 = -1.2.$

Case 3: Buy Neutral Straddle

- buy 86 call options ($\Delta_1 = 0.52$)
- buy 100 put options ($\Delta_2 = -0.45$)

$n_1/n_2 = -\Delta_2/\Delta_1 = -(-0.45)/0.52 = 0.865.$

In connection with the previous example, the next example presents results, which are calculated using Delta Hedging. The investment company has written 10 European call options on a stock and wishes to:

- hedge this portfolio to make its value insensitive to small changes in the stock price (equivalently, require portfolio's delta to be zero);
- make the portfolio self-financing (equivalently, make $0 net investment in a portfolio, which is created by going to long or short different securities including the riskless bond).

Part A	
Input data	Value
1. Stock price	$ 50
2. Strike price	$ 50
3. Time to Expiration	65 days
4.Volatility *	25 %
5. Risk-free rate**	6%

* A 365-days year is assumed.

** A 360-days year is assumed.

Part B				
Option price				
Value of option	Delta	Gamma	Vega	Theta
$ 2.3740	0.5620	0.0747	8.3158	–0.0203

The investment company writes (sells) 10 calls, the investing proceeds in m_S shares of the stock and B dollars in the riskless bond. Selling 10 calls (each contract is on 100 shares) gives

$$10 \times (100 \times \$2.3740) = \$2374.$$

The value of the initial portfolio is zero:

$$-2374 + m_S 50 + B = 0.$$

Portfolio delta = call delta + stock delta. Thus,

$$\delta_p = -10\times(100\times0.5620) + m_S = 0.$$

Solving two linear equations in two unknowns:

$$m_S = 562,$$
$$B = 2374 - 562 \times 50 = -\$25\,726.$$

To summarize:

• Sell 10 calls and receive $2374.

• Buy 562 shares of stock at a cost of $28 100.

• Remainder $25 726 (28 100 – 2374 = $ 25 726) is borrowed to finance stock purchase.

• Portfolio net value as well as delta is zero.

When the delta neutral portfolio is constructed, the following problems with Delta hedging appear:

• The portfolio is designed to be delta neutral. Yet, its value changes, even when the stock price remains unchanged.

• Although the portfolio is designed under continuous rebalancing assumption, it is not revised for a day. This generates a hedging error.

• Moreover, the portfolio already is not to be self-financing. It will now require a cash inflow or outflow to rebalance it to a delta neutral position. [1, 5, 9, 19]

The following example is used to analyse the problems with Delta hedging:

Stock price (in $)	Option price (in $)	Value of option position (in $)	Value of stock Position (in $)	Value of portfolio (in $)
48.00	1.3811	–1 384.40	26 976.00	–138.73
49.00	1.8303	–1 830.30	27 538.00	–22.59
49.50	2.0824	–2 082.40	27 819.00	6.31
50.00	2.3536	–2 353.60	28 100.00	16.11
50.50	2.6437	–2 643.70	28 381.00	7.01
51.00	2.9521	–2 952.10	28 662.00	–20.39
52.00	3.6208	–3 620.80	29 224.00	–127.09
Volatility* 25%				
Risk-free rate** 6%				

* A 365-days year is assumed.
** A 360-days year is assumed.

Black-Scholes model assumes constant volatility. In reality, volatility can change sometimes fairly quickly. This is another model misspecification. Suppose that the volatility increases to 30% from 25 %. The reason for this is a change in the value of the liability. Our hedged portfolio incurs substantial losses. This loss is due to “volatility” risk, a well-know hazard. The following table presents changes in hedged portfolio due to volatility changes:

Part A *Volatility Increase*				
Stock Price (in $)	Option Price (in $)	Value of option position (in $)	Value of stock position (in $)	Value of portfolio (in $)
48.00	1.777	–1 777.00	26 976.00	–531.29
49.50	2.4950	–2 495.00	27 819.00	–406.29
50.00	2.7665	–2 766.50	28 100.00	–396.79
50.50	3.0538	–3 053.80	28 381.00	–403.09
52.00	4.0054	–4 005.40	29 224.00	–511.69
Volatility* 30%				
Risk-free rate** 6%				
Time to Expiration – 64 days				

* A 365-days year is assumed.

** A 360-days year is assumed.

II. 2. Construction of the gamma neutral portfolio

Delta neutral hedge has gamma risk, which needs to rebalance when Δ changes.

Γ (gamma) measures sensitivity of Δ to large stock price. Gamma hedge requires another hedging option:

$$\theta = r \times \Delta + \Gamma \textit{ neural portfolio value},$$

where r is the current continuously compounded risk-free interest rate.

Example No 6. The investor has 1 share and –5.1917 calls with X=100 and 5.0251 calls with X = 110. Thus,

$$\Delta_p = 1 + [-5.1917 \times 0.6151] + [5.0251 \times 0.4365] = 0,$$

$$\Gamma_p = 0 + [-5.1917 \times 0.0181] + [5.0251 \times 0.0187] = 0.$$

Immune to small and large price changes requires rebalancing over time and is vulnerable to σ changes [7, 13].

II. 3. Construction of the vega neutral portfolio

Another type of portfolio hedging is the so called “vega” hedging. It is related to the decrease in the extent of volatility (standard deviation) and this targets at reduction of the risk expositions values. In this relation in order to construct a vega neutral portfolio, the rate of the vega coefficient of the considered portfolio must be equal to 0.

In the securities portfolios management this is most often done changing the number of options, covered by the portfolio. In constructing a vega neutral portfolio, the availability of a short-term position is required with one option and one long-term with another option from the very beginning [2, 19].

Example No 7. The next example presents Vega Hedging. The input data are the following:

Coefficient	Call option 1	Call option 2	Stock
Delta	$\Delta_{Call1} = 0.6$	$\Delta_{Call2} = 0.6$	$\Delta_{stock} = 1.0$
Gamma	$\Gamma_{Call1} = 0.5$	$\Gamma_{Call2} = 0.7$	$\Gamma_{stock} = 0.0$
Vega	$\nu_{Call1} = 1.5$	$\nu_{Call2} = 1.2$	$\nu_{stock} = 0.0$

Case 1: Delta Hedging with Call option 1

• By definition of Δ_{Call1} = 0.6. This is one short American call contract (100 shares) and long 60 shares of stock.

Compute Delta = 60×1 – 100(0.60) = 0 (Neutral)

Compute Gamma = 60×0 – 100(0.50) = –50 (risky)

Compute Vega = 60×0 – 100(1.50) = –150 (risky)

Case 2: Make Gamma/Vega neutral by adding Call option 2

• Buy 50 Call 2's to create Gamma neutrality

Gamma = –50 (Case1) + 50(1) (Call2) = 0

(Notice that this changes initial Delta: 0 + 50(0.7) = 35)

• Now, sell 35 shares to eliminate Delta exposure:

New Gamma = 0 = 0 – 35(0)

New Delta =0 = 35 – 35(1)

Final Portfolio: 15 Shares, 100 short Call option 1 and 50 Long Call option 2.

II. 4. Construction of the "delta–gamma–vega" neutral portfolio

Very often a combination of gamma and vega (ν) hedging is used. The main characteristics are:

• Δ can also change;

• ν measures sensitivity of option price to σ;

• Two more assets are needed to hedge Γ and ν.

Example No 8. This example analyzes the following data:

• Asset: Δ neutral portfolio; Γ = –5000; ν = –8000;

• Option 1: Δ = 0.6; Γ = 0.5; ν = 2.0;

• Option 2: Δ = 0.5; Γ = 0.8; ν = 1.2;

• Portfolio of 3240 of asset, 400 of option 1, and 6000 of option 2 has zero Δ, Γ, ν.

The portfolio needs rebalancing due to time decay (hedge with θ) .

In practice the various traders and investors have different preferences. Some want to construct "delta-neutral" portfolios. Others wish to protect their options portfolio from big changes in the base assets prices and thus to create a portfolio, whose values both of delta and gamma are equal to 0 or the so called "gamma-neutral" portfolio. Third want to protect their portfolio against small changes in the value of the standard deviation from the base asset as addition to the "delta and gamma" neutral portfolios and this is the so called "delta-gamma-vega" neutral portfolio.

Using the Black-Scholes model for European options, this example creates an equity option portfolio that is simultaneously delta, gamma, and vega neutral. The value of a particular greek of an option portfolio is a weighted average of the corresponding greek of each individual option. The weights are the quantity of each option in the portfolio. Hedging an option portfolio thus involves solving a system of linear equations, an easy process in MATLAB [13, 15, 35].

Let us analyze the following example, applying MATLAB.

Example No 9. For analysis of this example it is assumed that the annualized risk-free rate is 10 % and is constant for all maturities of interest. The arbitrary portfolio value is $21000 and solves the linear system of equations such that the overall option portfolio is simultaneously delta, gamma, and vega-neutral. The input data are presented in the following table:

Stock price (in $)	Strike price (in $)	Time to expiration (years)	Volatility σ	Dividend rate %	Type option
100	100	0.2	0.3	0	Call
119	125	0.2	0.2	0.025	Put
87	85	0.1	0.23	0	Call
301	315	0.5	0.25	0.0333	Put

Finally, compute the market value, delta, gamma, and vega of the overall portfolio as a weighted average of the corresponding parameters of the component options.

You can verify that the portfolio value is $21,000 and that the option portfolio is indeed delta, gamma, and vega neutral, as desired. Hedges based on these measures are efficient only for small changes of the underlying variables.

Option No	Price option (in $)	Delta	Gamma	Vega
1	6.3441	0.5856	0.0290	17.4293
2	6.6035	–0.6255	0.0353	20.0347
3	4.3993	0.7003	0.0548	9.6837
4	23.6694	–0.4830	0.0074	84.5225

Thus, for the portfolio:

Portfolio value	$ 21 000 .00
Portfolio Delta	0.00
Portfolio Gamma	–0.00
Portfolio Vega	0.00

Example No 10. The prtfolio consists of eight instruments: two bonds, one bond option, one fixed rate note, one floating rate note, one cap, one floor, and one swap. Both hedging functions require some common inputs, including the current portfolio holdings (allocations) and a matrix of instrument sensitivities. To create these inputs:

No of securities	Price (in $)	Part in portfolio	Delta	Gamma	Vega
1	98.72	100	–272.65	1030.00	0.00
2	97.53	50	–347.43	1622.69	–0.04
3	0.05	–50	–8.08	643.40	34.07
4	98.72	80	–272.65	1029.90	0.00
5	100.55	8	–1.04	3.31	0.00
6	6.28	30	294.97	6853.56	94.69
7	0.05	40	–47.16	8459.99	93.69
8	3.69	10	–282.05	1059.68	0.00

The current portfolio sensitivities are a weighted average of the instruments in the portfolio. The function **targetSens = holdings’** × **Sensitivities** :

Coefficient	Delta	Gamma	Vega
Portfolio	–62 200.22	79 046.21	5 852.91

The function **hedgeopt** is used to determine the minimum cost of hedging a portfolio given a set of target sensitivities. When the function was used, portfolio target sensitivities are treated as equality constraints during the optimization process. To illustrate the use of **hedgeopt,** it is supposed that the existing portfolio must be maintained. The first form of hedgeopt minimizes the cost of hedging a portfolio given a set of target sensitivities. The existing portfolio composition and exposure should be able to do so without spending any money. To verify this, set the target sensitivities to their current. The portfolio composition and sensitivities are unchanged, and the cost associated with doing nothing is zero. The cost is defined as the change in the portfolio value. This number cannot be less than zero because the rebalancing cost is defined as a nonnegative number. If Value0 and Value1 represent the portfolio value before and after rebalancing, respectively, the zero cost can also be verified by comparing the portfolio values. Thus, Value0= Value1 =24674.62.

Building upon the previous example, it is assumed the cost to achieve an overall portfolio dollar sensitivity of [–25000 –3500 3000], while allowing trading only in securities 2, 3, and 6 (holding positions of securities 1, 4, 5, 7, and 8 fixed.) To find the cost, first set the target portfolio dollar sensitivity. Finally, call **hedgeopt** and again examine the results.

Coefficient	Delta	Gamma	Vega
Portfolio	–25 000	–3 500	3 000

After this the part of securities in the portfolio was changed for the second position (–141.03); for the third (137.26) and for the sixth (–57.96). This change will be reflected in the value of the portfolio and it is 19974.02. Recomputing Value1, the portfolio value after rebalancing, Value1 = 4700.60.

As expected, the cost of $19974.02, is the difference between Value0 and Value1, $24674.62 – $4700.60. Only the positions in securities 2, 3, and 6 have been changed.

The above example illustrates a partial hedge, but perhaps the most interesting case involves the cost associated with a fully-hedged portfolio (simultaneous delta, gamma, and vega neutrality). In this case, set the target sensitivity to a row vector of zeros and call **hedgeopt** again. At computing these values, the part of the securities in the portfolio must be changed, for the second position to –182.36; for the third 19.55 and for the sixth –32.97. This change will be reflected in the value of the portfolio and it is $ 24055.90. The new value of the portfolio Value1= 618.72.

Assume, for example, that it is necessary to spend as much as $40000 and the intention is to see what portfolio sensitivities will result along the cost frontier. Assume the same instruments are held fixed and that the cost frontier is evaluated from $0 up to $40000 at increments of $1000. In Fig. 1 are illustrate a rebalancing cost profile and in Fig. 2 are illustrate funds available for rebalancing.

Example No 11. The table lists the portfolio of OTC Euro options on a security:

Type option	Value of securities (in $)	Delta	Gamma	Vega
Call	–1000	0.5	2.2	1.8
Call	–500	0.8	0.6	0.2
Put	–2000	–0.4	1.3	0.7
Call	–500	0.7	1.8	1.4

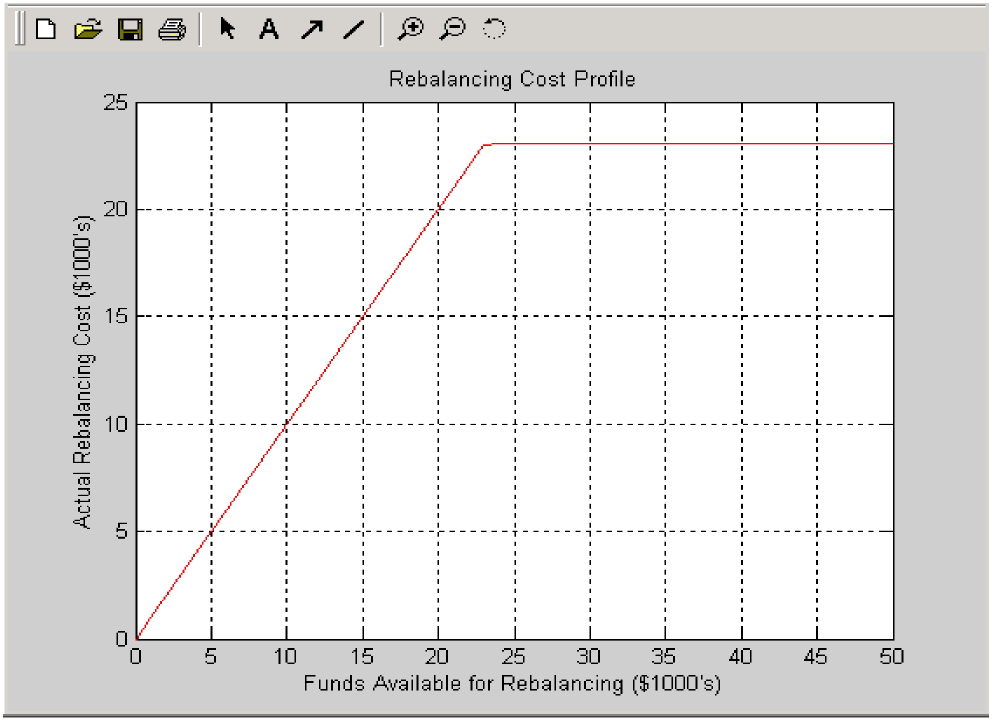

Fig. 1. Rebalancing cost profile

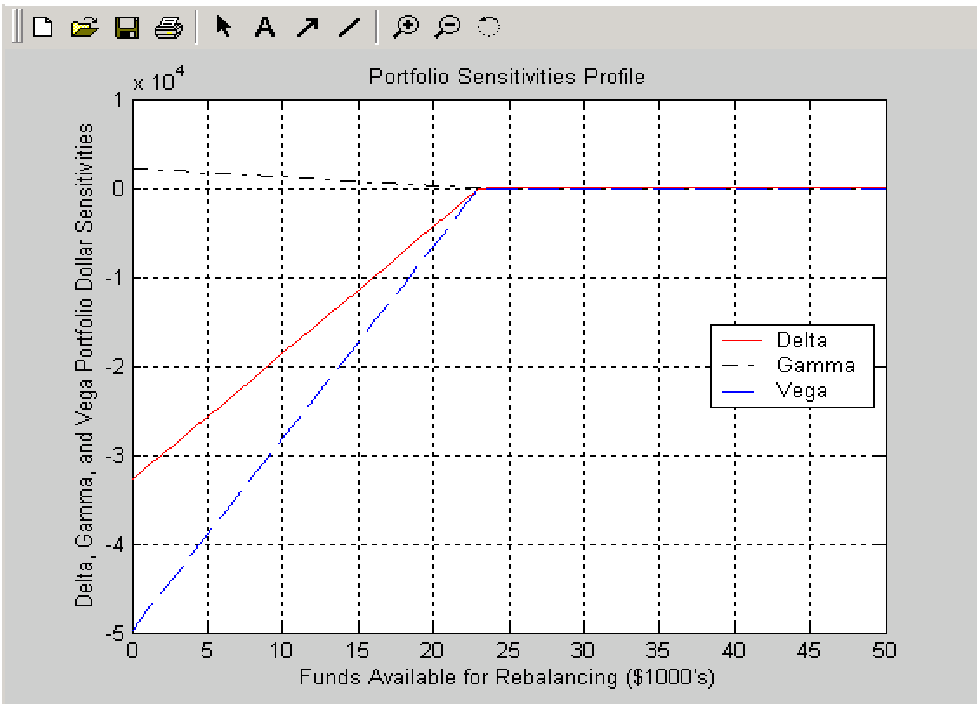

Fig. 2. Funds available for rebalancing

This translates into the following portfolio Greeks:

Type option	Value of securities (in $)	Delta	Gamma	Vega
Call	–1000	–500	–2200	–1800
Call	–500	–400	–300	–100
Put	–2000	800	–2600	–1400
Call	–500	–350	–900	–700
Portfolio		–450	–6000	–4000

To summarize:
• *Portfolio delta = –450.*
Investor should be long 450 contracts.
• A traded option is available with Δ=0.6, Γ=1.5 and Λ=0.8.
Delta- and gamma-neutral portfolio is obtained at currency position of –1950 and position in traded option of 4000.
• To make the portfolio both delta- and vega-neutral a currency position of –2550 and position in traded option of 5000 is needed [25, 26].

II. 5. Construction of the "delta-gamma-theta-vega-rho" neutral portfolio

Greeks based strategies are opened and maintained in order to attain a specific level of sensitivity. Mostly, these strategies are set to attain zero sensitivity.

Example No 12. The underlying asset is the S&P100 stock index. The options on this index are European. Note that the stocks in the index pay dividends that affect the options prices.

Input data	
1. Stock price *S*	$300
2. Strike price *X*	$300
3. Time to Expiration *T*	1 year
4.Volatility σ	18 %
5.Risk-free rate *r*	8%
6. Dividend rate *q*	3 %
7. Call option price	$28.25

The data for the sensitivities are:

Call Option				
Option price	Delta	Gamma	Vega	Rho
$ 28.25	0.6245	0.0067	0.0109	0.0159

The constructed Delta-Gamma-Vega-Rho-neutral portfolio is as follows:

Call	0	1	2	3
X	300	305	295	300
T (days)	365	90	90	180
Volatility	18 %	18 %	18 %	18 %
r	8 %	8 %	8 %	8 %
Dividends	3 %	3 %	3 %	3 %
Price call option	$ 28.25	$ 10.02	$ 15.29	$ 18.59

After the analysis the delta-gamma-vega-rho-neutral portfolio must be again constructed:

Call	Delta	Gamma	Vega	Rho
0	0.6245	0.0067	0.0109	0.0159
1	0.4952	0.0148	0.0059	0.0034
2	0.6398	0.0138	0.0055	0.0044
3	0.5931	0.0100	0.0080	0.0079
Neutrality	1.0	0.0	0.00	0.0

In order to neutralize the portfolio to all risk exposures, following the sale of the initial call,the portfolio's proportions of the other calls and the stock index are determined such that all the portfolio sensitivity parameters are zero simultaneously [10, 34].

Thus, $\Delta = \Gamma = = \theta = \upsilon = \rho = 0$ **simultaneoulsy**.

Therefore:

Delta = Δ = 0 –

$$n_S + n_0(0.6245) + n_1(0.4952) + n_2(0.6398) + n_3(0.5931) = 0;$$

Gamma = Γ = 0 –

$$n_0(0.0067) + n_1(0.0148) + n_2(0.0138) + n_3(0.0100) = 0;$$

Vega = υ = 0 –

$$n_0(0.0109) + n_1(0.0059) + n_2(0.0055) + n_3(0.0080) = 0;$$

Rho = ρ = 0 –

$$n_0(0.0159) + n_1(0.0034) + n_2(0.0044) + n_3(0.0079) = 0.$$

Thus, the short position is the call 0, i.e., $n_0 = -1$ and after this the simultaneous equations are solved.

The solution is:

$n_0 = -1.000$	short call No 0
$n_1 = 0.840$	long 0.840 call No 1
$n_2 = -1.900$	short 1.900 call No 2
$n_3 = 2.040$	long 2.040 call No 3
$n_S = 0.2120$	long 0.212 of the index

To see what this solution means in practical terms, multiply all the weights by 10 000. The portfolio becomes:

Short 100 CBOE calls No 0;
Long 84 calls No 1;
Short 190 calls No 2;
Long 204 calls No 3;
Long 2120 units of the index.

Every index unit is $100, so buying $212 000 is worth for the index.

Another delta-gamma-vega-rho- neutral portfolio was constructed if S increases from $300 up to $310, r increases from 8% to 9% and σ increases from 18% to 24%. The new data for the portfolio is:

Portfolio	Initial value	New value	Change
– 1.0(#0)	– $28.25	– $42.81	– $14.56
(0.212)S	$63.60	$65.72	$2.12
(840)#1	$8.40	$16.42	$8.02
(–1.9)#2	– $29.05	– $48.97	– $19.92
(2.04)#3	$37.97	$62.20	– $24.25
			Error– $ 0.09

III. Concluding remarks

The most straightforward form of hedging is a buy-and-hold strategy. This involves finding the optimal mix of cash and equity which will, for example, minimize the variance of the hedging error at the maturity date of the derivative. However, this method can only reduce the risk by a small amount. To improve it the invesors must consider dynamic portfolio strategies, particularly ones where the optimal mix of cash and equity depends on the state of the market at given moments in future.

On the basis of the examples mentioned, in relation to the application of the Greek letters for hedging, definite conclusions can be drawn and frequently met problems to be indicated.

The approach is referred to as *Delta hedging*. Delta hedging works well if:

• using the correct model (for example, the Black-Scholes-Merton model) for S with the correct values for r and σ;

• able to rebalance the portfolio continuously;

• there are no transaction costs.

An example of the effect of discrete-time rebalancing is given in Fig. 3 for an European call option. The plots present the correct volatility for pricing and hedging. Approximately, the surplus (accumulated hedging error at T) is centred around zero. The plots show histograms of the hedging errors based on 1000 independent simulations. Comparison of the plots shows the effect of the time between rebalances and of incorrect estimation of the volatility. S = $ 100, μ = 0.08, r = 0.06, T = 0.5, X = $ 100, true σ = 0.2. The correct value for σ (0.2) and an incorrect value for σ (0.15) have been used for pricing and hedging.

The plots show the impact of underestimating the volatility (for both pricing and hedging). First, we can see that the average surplus is now negative, because we have undercharged. Second, the standard deviation of the errors is larger. Third, rebalancing more frequently does not reduce significantly the standard deviation because the wrong σ has been used [4, 12].

Thus two sources of a hedging error have been identified:

• due to discrete-time rebalancing;

• due to errors in the estimated parameters.

The plots shown in Fig. 4 (left) are presented in a different way in Fig. 4 (right) in order to show some of this dependency. The hedging errors have tended to be larger when the final share-price is close to the option strike price of $ 100. This is because σ is most sensitive to changes in S (largest errors as noted above). The parameter t is close to T and S is close to the strike price X. In Fig. 4, S = $ 100, μ = 0.08, r = 0.06, T = 0.5, X = 100, true σ = 0.2. The example shows how the size of hedging errors is related to the final share price, S. The plot shows results for 4-day rebalancing and on the right – 1-day rebalancing.

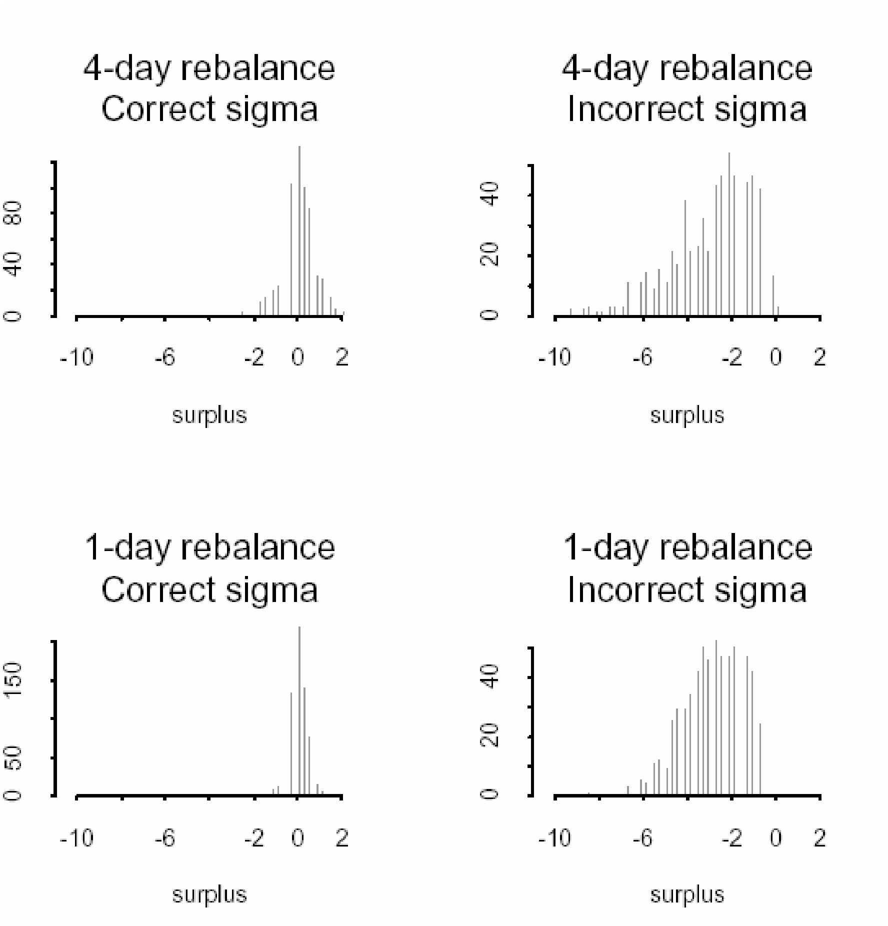

Fig. 3. Hedging errors for an European call option

A simple example showing the impact of Gamma hedging on hedging errors is indicated in Fig. 5. The plot (see also Fig. 4) shows hedging errors when there is hedged portfolio every 4 days using cash plus *S*. The plot is conctructed using a combination of cash, shares and a new, exchange-traded put option for hedging the portfolio. In Fig. 6, $S = \$100$, $\mu = 0.08$, $r = 0.06$, $T = 0.5$, $X = \$100$, true $\sigma = 0.2$. The plot shows no Gamma hedging and Gamma- neutral portfolio consisting of one European call option $T = 0.5$, $X = \$100$ plus a variable number of units of an exchange-traded European put option with $T = 1$ and $X = \$ 105$.

With Vega hedging we aim to invest in a mixture of shares and derivatives which make the total portfolio *Vega neutral*: that is the sum of the Vega's for the individual investments is zero. If the assets are enough (exchanged-traded derivatives) this can be combined with Gamma and Delta hedging.

Vega hedging measures the impact of the parameter errors on derivative pricing. If the Vega is large then there may be significant bias in prices. However, simulations show that Vega hedging has only a small effect on the performance of hedging strategies: it removes some of the bias in the mean surplus but does not significantly reduce its variance [12].

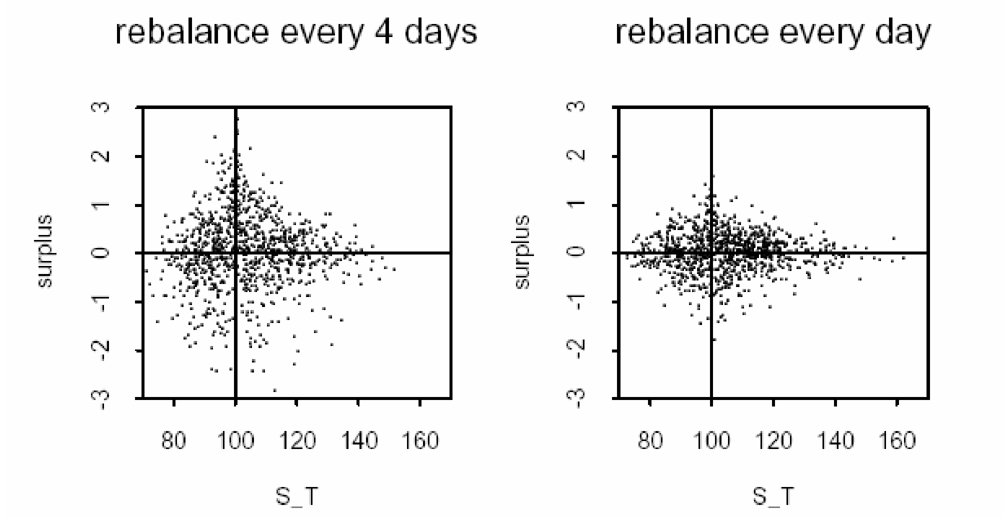

Fig. 4. Hedging errors for a European call option based on 1000 simulations

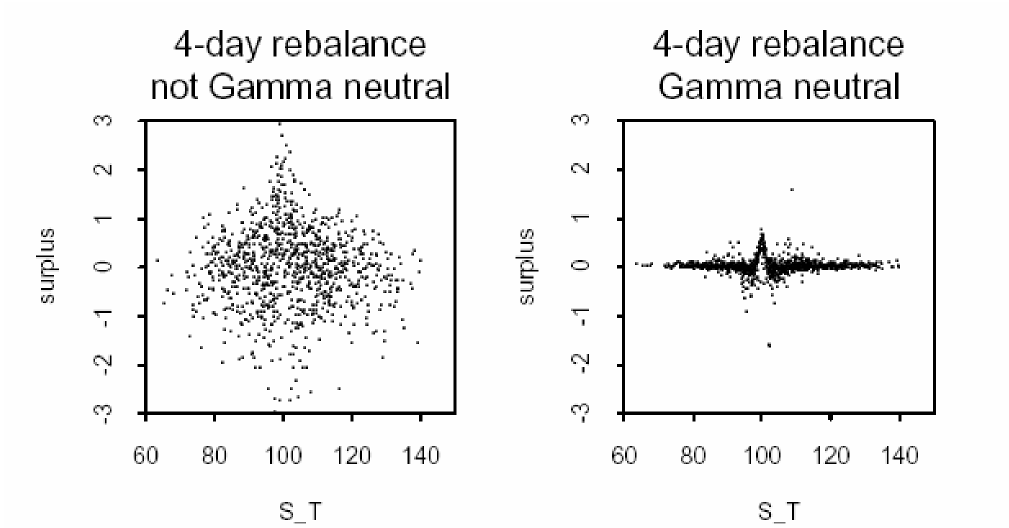

Fig. 5. Hedging errors for an European call option based on 1000 simulations with rebalancing every 4 days

In practice an investor will want to look at their portfolio of derivatives as a whole rather than individually. They will regularly calculate the portfolio total Delta, Gamma and Vega. They will aim to keep the portfolio Delta neutral most of the time. Maintaining full Gamma and Vega neutrality is more difficult because of the costs of trading and the relative lack of a deep market in appropriate traded derivatives. However, if these deviate too far from zero then they will need to rebalance. If the portfolio is far from neutral then this might even mean that the investor should dispose some of its derivative liabilities in order to reduce their exposures to certain types of risk.

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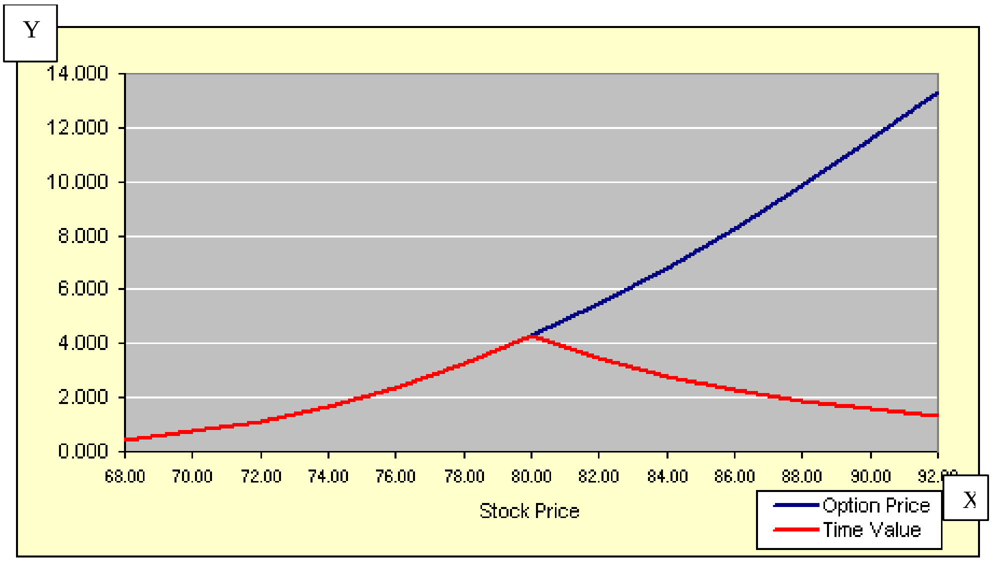
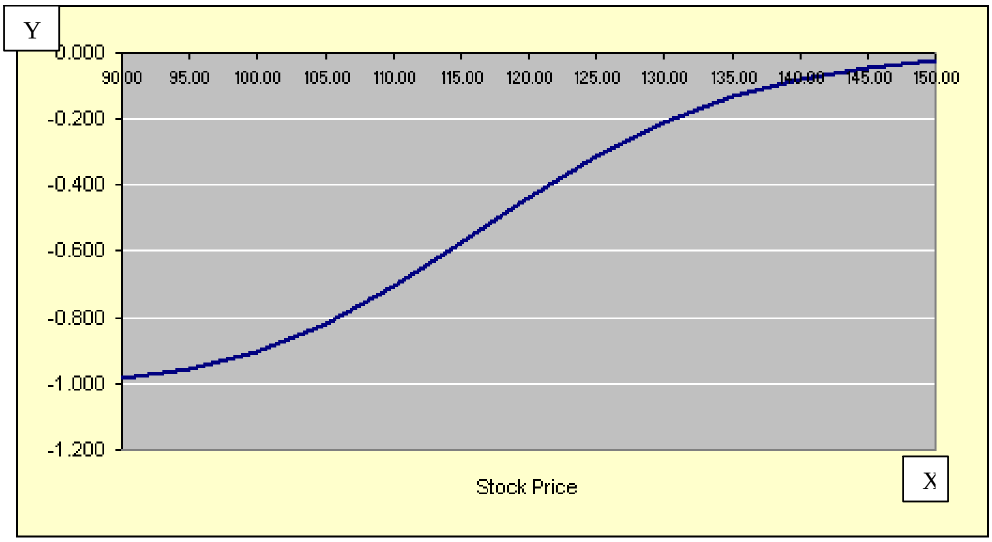
Appendix No 1



Call option price & time value by stock price

Description:
Let X = Stock price;
Let Y = Option Price.

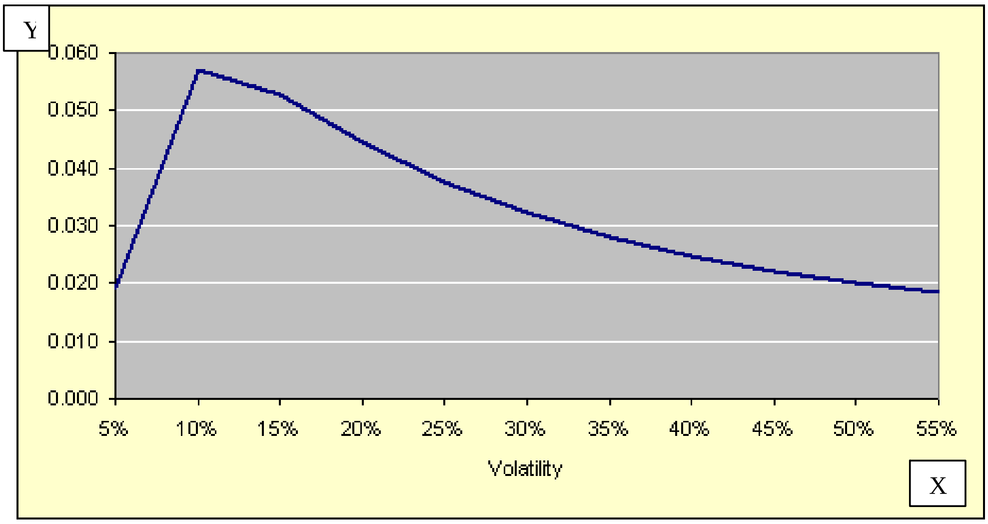
Appendix No 2



Put option delta by stock price

Description:
Let X = Stock price;
Let Y = Delta.

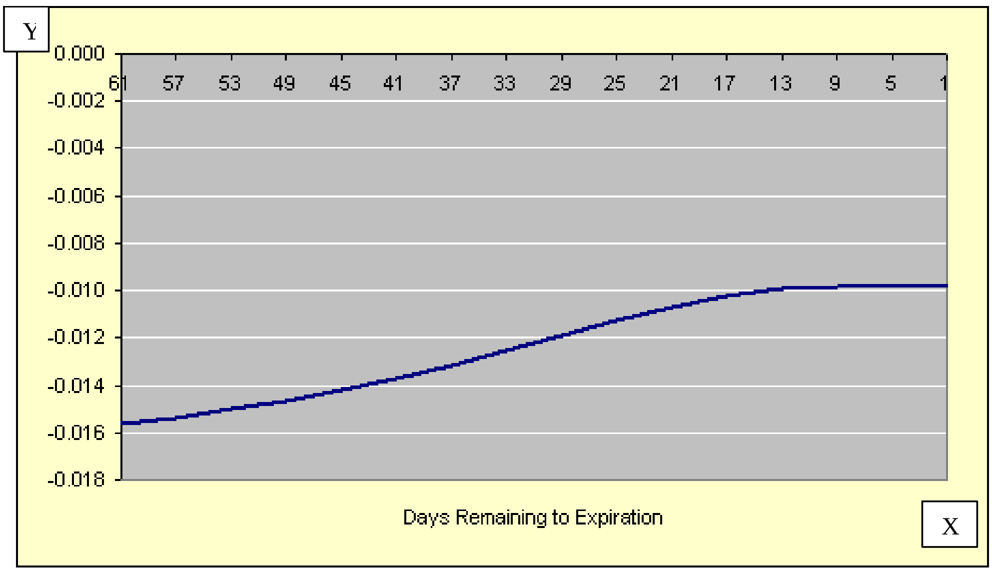
Appendix No 3



Put option gamma by volatility

Description:
Let X = Volatility;
Let Y = Gamma.

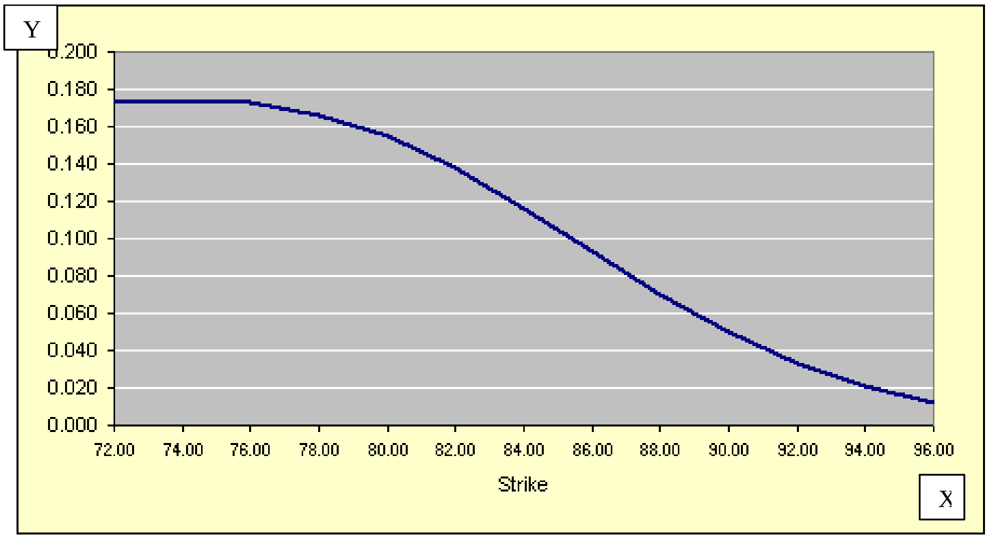
Appendix No 4



Call option theta by time remaining to expiry

Description:
Let X = Time Remaining to Expiry;
Let Y = Theta.

Appendix No 5



Call option rho by strike

Description:
Let X = Strike price;
Let Y = Rho.