Experimental Study of Lyapunov Equation Solution Bounds for Power Systems Models

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Abstract: This research work investigates the applicability of some lower and upper, matrix and scalar bounds for the solution of the Continuous Algebraic Lyapunov Equation (CALE), when the coefficient matrices are the state matrices of real data models. The bounds are illustrated for two different models describing the dynamic behavior of power systems – a two-area power system and an interconnected power system. Some important conclusions referring to the accuracy of the respective estimates are made, as well.

Keywords: Lyapunov equation, solution bounds, power systems.

1. Introduction

The problem of deriving bounds for the solution of the Continuous Algebraic Lyapunov Equation (CALE) attracts interest for more than half a century. This is due to both theoretical and practical reasons. In some cases, due to its high order, the direct solution of this equation is impossible, and in others it is sufficient to have at disposal only some estimates for it. The main difficulty arises from the fact, that the available upper bounds are valid under some assumed restrictions imposed on the coefficient matrix. Due to this, valid solution bounds are possible only for some special subsets of negative stable (Hurwitz) coefficient matrices.

The main purpose of this research is to investigate the quality of some available lower and upper, matrix and scalar bounds for the solution of the CALE. Two state space models with real data of a relatively high order $n$ of power systems
are used as test examples. The first one is a model of a two-area power system with 
\( n = 11 \) states and the second one describes the dynamics of an interconnected power 
system with four local subsystems and \( n = 8 \) states. Lower and upper matrix, 
eigenvalue and trace estimates are obtained for the solution. They are compared 
with the exact values and some important conclusions referring to their accuracy are 
drawn.

The following notations will be used: \( A \geq 0 \) (\( A \geq 0 \)) indicates that \( A \) is a 
positive (semi-positive) definite matrix; \( A \in \mathbb{R}_{n,m} \) (\( \mathbb{R}_{n,n} = \mathbb{R}_n \)) and \( a \in \mathbb{R}^n \) denote a 
real \( n \times m \) matrix \( A \) and \( n \times 1 \) vector \( a \); \( \text{tr}(A) \), \( A^{1/2} \), \( A^{-1} \), \( A^T \) are the trace (sum of 
eigenvalues), the square root (if \( A \) is positive semi-definite), the inverse (if \( A \) is 
nonsingular) and the transpose of matrix \( A \); the symmetric part of matrix \( A \) is 
denoted \( A_s = A^T + A \); \( \lambda_{\min}(A) \) and \( \lambda_{\max}(A) \) denote the minimal and maximal 
eigenvalue of a symmetric matrix \( A \) respectively; the identity matrix of respective 
dimension is \( I \).

2. Bounds for the solution of the CALE

From Lyapunov’s stability theorem it follows that a matrix \( A \) is Hurwitz (negative 
stable) if and only if the CALE 

(1) \[ A^T P + PA = -Q, \quad A \in \mathbb{R}_n \]

has a unique positive definite solution matrix \( P \) for any given positive definite 
matrix \( Q \). Due to various reasons, having at disposal only bounds on \( P \) can be 
sufficient in some practical control design and analysis problems.

It must be emphasized on the fact that the upper bounds are always more 
difficult to derive, since they are valid only under some restrictions. Till 2004 all 
available upper bounds for the solution of the CALE (e.g., [1] and [2]) are valid 
under the very conservative condition that the symmetric part of the coefficient 
matrix \( A \) is negative definite. By making use of the singular value decomposition of 
\( A \), i.e.,

\[ A = UV \Sigma V^T = (U \Sigma U^T)V V^T = (AA^T)^{1/2} F, \]

\[ UU^T = V V^T = FF^T = I, \quad \Sigma = \text{diag}(\sigma_i) > 0, \quad i = 1,...,n, \]

it was proved in [5] that

(2) \[ A_i < 0 \Rightarrow F_i = A_i^T R + RA_i < 0, \quad R = (AA^T)^{-1/2}. \]

This simple fact helped to extend the set of stable coefficient matrices for 
which there exist valid upper bounds. Various types of based on this result bounds 
were derived in [3] and [4].

**Definition 1.** A matrix \( L > 0 \) is said to be a Lyapunov Matrix (LM) for \( A \) if 
\( A^T L + LA < 0 \).

In this sense, if the symmetric part of \( A \) is negative definite, then \( I \) and \( R \) are 
LMs for \( A \), in accordance with (2), but \( R \) can be a LM even if this condition does 
not hold.

If there exist positive definite matrices \( P, P_u \), such that
\[ A^2 P_a + P_a A \leq -Q \leq A^2 P_t + P_t A \Rightarrow 0 < P_t \leq P \leq P_a, \]
i.e., \( P_t, P_a \) are lower and upper matrix bounds for \( P \), respectively. In other words, the solution estimation problem can always be solved if such matrices can be obtained. On the other hand, their determination in the general case, e.g., by LMI solution, may require computational effort comparable with the one needed for the direct solution of the Lyapunov Equation (LE) and therefore should be avoided.

**Theorem 1.** Let \( R \) in (2) be a LM for \( A \). Then the solution \( P \) of the LE (1) has the following bounds:

\[ \begin{align*}
\mu_l &\leq P_t \leq P_a = \mu_u, \\
\lambda_m(P) &\leq \lambda_m(P) \leq \lambda_m(P) \leq \lambda_m(P), \quad i = 1, \ldots, n, \\
\text{tr}(P) &\leq \text{tr}(P) \leq \text{tr}(P). 
\end{align*} \]

**Proof:** Let the above assumption holds, i.e., the symmetric part of matrix \( F \) is negative definite. From the definition in (4) of the scalar and matrix parameters \( \mu_l, \mu_u, P_t, P_a \), it follows that

\[ \begin{align*}
\mu_l &\leq \lambda_m([-F_1]^{1/2} Q([-F_1]^{1/2}) \Rightarrow \mu_l I \leq \lambda_l([F_1]^{1/2} Q([F_1]^{1/2}) \Rightarrow \mu_l I \leq \lambda_l(F_1), \\
\mu_u &\geq -\lambda_l([F_1]^{1/2} Q([-F_1]^{1/2}) \Rightarrow -\mu_u I \geq -\lambda_l(F_1), \\
\text{tr}(X) &\leq \text{tr}(Y) \leq \text{tr}(Z) \quad \text{for arbitrary } n \times n \text{ symmetric matrices } X, Y \text{ and } Z. 
\end{align*} \]

3. Power systems models

Large-scale power systems consist of a number of interconnected via tie-lines power control areas. Different complicated nonlinear models of such a system are available. However, for control design purposes a simplified linearized model is usually used. A two-area power system is taken as a test system in this study. The generators are assumed to be a coherent group in each area, which includes a governor, a reheater and a steam turbine.

3.1. Two-area power system

The set of first order differential equations with constant coefficients governing the overall process in a two-area power system and the description of the respective variables and coefficient parameters is given below [6]:

\[ \begin{align*}
\dot{P}_{c1} &= k_{i1} b_i F_i + k_{i1} P_{se}, \\
\dot{X}_{g1} &= -(1/t_{g1}) P_{c1} - (1/t_{g1}) X_{g1} - (1/r_{g1}) F_i + (1/t_{g1}) u_{21},
\end{align*} \]
\[ \begin{align*}
\dot{P}_{R1} &= (1/t_{R1})X_{G1} - (1/t_{R1})P_{R1}, \\
\dot{P}_{R1} &= (k_{R1}/t_{R1})X_{G1} + aP_{R1} - (1/t_{R1})P_{R1}, \\
\dot{F}_i &= (k_{p1}/t_{p1})P_{R1} - (1/t_{p1})F_i - (k_{p1}/t_{p1})P_{ue} - (k_{p1}/t_{p1})u_{i1}, \\
\dot{P}_{ue} &= 2\pi T_{G1}F_i - 2\pi T_{G1}F_2, \\
\dot{P}_{C2} &= -k_{i2}F_i - k_{i2}P_{T2}, \\
\dot{X}_{G2} &= -(1/t_{G2})P_{C2} - (1/t_{G2})X_{G2} - (1/r_{G2}P_{T2})F_2 + (1/t_{G2})u_{22}, \\
\dot{P}_{T2} &= (1/t_{T2})X_{G2} - (1/t_{T2})P_{T2}, \\
\dot{F}_2 &= (k_{p2}/t_{p2})P_{ue} - (1/t_{p2})F_2 + (k_{p2}/t_{p2})P_{T2} - (k_{p2}/t_{p2})u_{22},
\end{align*} \]

where

- \( P_{Ci} \) – incremental change in the integral controller \( i \),
- \( X_{Gi} \) – fictitious state variable,
- \( P_{Ri} \) – incremental change in the output energy of the \( i \)-th reheat type turbine in MW,
- \( P_{ni} \) – incremental change in the output of the \( i \)-th subsystem in MW,
- \( F_i \) – incremental frequency deviation in subsystem \( i \) in Hz,
- \( P_{ui} \) – incremental change in the tie-line power,
- \( t_{Gi} \) – \( i \)-th governor time constant in s,
- \( t_{ni} \) – \( i \)-th subsystem time constant in s,
- \( t_{ri} \) – \( i \)-th reheat time constant in s,
- \( t_{pi} \) – \( i \)-th subsystem time constant in s,
- \( k_{pi} \) – \( i \)-th subsystem gain in Hz/MW
- \( k_{ji} \) – \( i \)-th subsystem integral control gain,
- \( b_i \) – \( i \)-th subsystem frequency-biasing factor in MW/Hz
- \( k_i \) – ratio between the output energy of the \( i \)-th turbine to the total output energy,
- \( r_i \) – speed regulation for the \( i \)-th subsystem due to the governor action in Hz/MW,
- \( t_{ij} \) – synchronizing coefficient of the tie-line power between subsystems \( i \) and \( j \).

The nominal values of the parameters are given in Appendix 1. Having in mind the above set of differential equations and defining the state and control vectors \( x(t) \) and \( u(t) \), respectively, as follows

\[ x(t)^T = (P_{C1} \ X_{G1} \ P_{R1} \ P_{ni} \ F_i \ P_{ue} \ P_{C2} \ X_{G2} \ P_{R2} \ P_{T2} \ F_2), \]

\[ u(t)^T = (u_{i1} \ u_{12} \ u_{21} \ u_{22}) \]
helps to put the model in the well known compact matrix (state space) form
\[ \dot{x}(t) = Ax(t) + Bu(t), \quad x(t) \in \mathbb{R}^n, \quad x(0) = x_0, \quad u(t) \in \mathbb{R}^d. \]

It has been concluded that the nominal open loop coefficient matrix \( A \) is not stable, since it has eigenvalues with positive real parts. An optimal stabilizing state control law \( u(t) = -Kx(t) \) has been applied to the system, where \( K = -B^T R \) and \( R \) is the positive definite solution of the continuous algebraic Riccati equation \( A^T R + R A - P B B^T R = -I \). The stable close loop nominal system is \( \dot{x} = Ax, \quad A_\omega = A - BK \). The nominal systems parameters and the computed for them state matrix are given in Appendix 1.

3.2. Interconnected power system

Consider the linearized model of an interconnected power system comprised of \( N \) local generators described by the set of differential equations [7]:
\[ \Delta \dot{\omega}_i = -R_i^{-1} \Delta \omega_i + \sum_{j=1}^{N} Y_{ij} \Delta \delta_j, \quad Y_{ij} = Y_{ji}, \quad i = 1, 2, ..., N, \]
\[ \delta_i = \Delta \omega_i, \quad i = 1, 2, ..., N, \]
where \( \omega_i \) and \( \delta_i \), \( i = 1, 2, ..., N \), denote the angular speed and deviation of the \( i \)-th rotor, \( Y_{ij} \) is the transfer conductance between subsystems \( i \) and \( j \), \( Y_{ii} \) is the self conductance of the \( i \)-th generator and \( R_i \) is a specific coefficient associated with the \( i \)-th subsystem. Using the notation \( x = (x_1^T, x_2^T, ..., x_N^T)^T \in \mathbb{R}^{2N}, \)
\( x_i = (\Delta \omega_i, \Delta \delta_i)^T \in \mathbb{R}^2 \), the above model can be rewritten in a compact matrix form, where the state matrix has the following typical structure
\[
A = \begin{bmatrix}
A_{11} & A_{12} & \cdots & A_{1N} \\
A_{21} & A_{22} & \cdots & A_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
A_{N1} & A_{N2} & \cdots & A_{NN}
\end{bmatrix} \in \mathbb{R}^{2N \times 2N},
\]
\[
A_{ii} = \begin{bmatrix}
-R_i^{-1} Y_{ii} \\
1 & 0
\end{bmatrix}, \quad A_{ij} = \begin{bmatrix}
0 & Y_{ij} \\
0 & 0
\end{bmatrix} = A_{ji} \in \mathbb{R}_2.
\]

The considered system is comprised of \( N = 4 \) subsystems with the following nominal parameter values
\( R_i = 0.01, \ i = 1, ..., 4, \ Y_{11} = -2.2, \ Y_{22} = -2.4, \ Y_{33} = -2.6, \ Y_{44} = -2.8, \ Y_{12} = 0.5, \ Y_{13} = 0.3, \ Y_{14} = 0.2, \ Y_{23} = 0.6, \ Y_{24} = 1, \ Y_{34} = 0.1. \)

The nominal stable state matrix \( A \) is given in Appendix 2.
4. Numerical experiments

4.1. Two-area power system

It is required to determine lower and upper, matrix and scalar bounds for the solution \( P \) of the LE, where the coefficient matrix in (1) is \( A_c \). The solution \( P \) provides also some valuable information about the quality of the transient process, since with the close loop system an integral performance index can be associated of the form

\[
J = \int_0^\infty x^T(t)Qx(t)dt = x_0^TPx_0
\]

which can be bounded from below and above by respective estimates (if available), as well, i.e.,

\[
x_0^TPx_0 \leq J \leq J_u = x_0^TP_ux_0
\]

which helps to estimate \textit{apriori} the dynamic behavior of the system for any given non-zero initial state vector without solving the CALE.

Since the symmetric part of the coefficient matrix \( A_c \) is not negative definite (it contains zero diagonal entries), all upper bounds based on this assumption are not valid in this case. But \( R \) is found to be a LM for it and the bounds in (4), (5) and (6) are all valid.

The LE (1) is solved for \( Q = I \). The scalar parameters in (4) are computed as \( \mu_l = 0.5, \mu_u = 1.991 \). The comparison process includes the analysis of the extremal eigenvalues and the traces of the exact solution matrix \( P \) and their lower and upper estimates. All eleven eigenvalues (given in a descending order) and traces of the solution \( P \) and its lower and upper matrix bounds (4), are given in Table 1 and Table 2.

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>( P_1 )</th>
<th>( P_{exact} )</th>
<th>( P_u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>115.4130</td>
<td>178.4124</td>
<td>441.3163</td>
</tr>
<tr>
<td>2</td>
<td>15.2589</td>
<td>24.6369</td>
<td>58.3469</td>
</tr>
<tr>
<td>3</td>
<td>0.9310</td>
<td>1.8672</td>
<td>3.5598</td>
</tr>
<tr>
<td>4</td>
<td>0.3133</td>
<td>0.5402</td>
<td>1.1981</td>
</tr>
<tr>
<td>5</td>
<td>0.1189</td>
<td>0.1403</td>
<td>0.4546</td>
</tr>
<tr>
<td>6</td>
<td>0.1119</td>
<td>0.1323</td>
<td>0.4280</td>
</tr>
<tr>
<td>7</td>
<td>0.0502</td>
<td>0.0535</td>
<td>0.1920</td>
</tr>
<tr>
<td>8</td>
<td>0.0256</td>
<td>0.0341</td>
<td>0.0979</td>
</tr>
<tr>
<td>9</td>
<td>0.0221</td>
<td>0.0326</td>
<td>0.0844</td>
</tr>
<tr>
<td>10</td>
<td>0.0034</td>
<td>0.0082</td>
<td>0.0131</td>
</tr>
<tr>
<td>11</td>
<td>0.0061</td>
<td>0.0081</td>
<td>0.0232</td>
</tr>
</tbody>
</table>
If tr(X) is the trace of some \( n \times n \) positive definite matrix \( X \), then the parameter \( \bar{\lambda} = \text{tr}(X)/n \) is introduced to define the “average” eigenvalue of \( X \). It becomes clear that the lower eigenvalue and trace bounds for \( P \) are more precise than the upper ones, which is not surprising. The accuracy of the bounds depends entirely on the parameters \( \mu_u, \mu_l \), since \( \Delta P = P_u - P_l = \mu R \), \( \mu = \mu_u - \mu_l \).

The smaller is the difference, the tighter are all bounds, since the matrix interval containing the solution \( P \) becomes narrower.

Now, it will be interesting to get some additional estimates. Define the “average” matrix estimate as \( \bar{E} = 0.5(P_l + P_u) \). It is clear that \( P_l \leq E \leq P_u \), i.e., \( E \) belongs to the matrix interval which contains the exact solution \( P \). The estimate error matrix is \( \Delta = P_u - P_l \). The following results for the eigenvalues, traces and average eigenvalues of these matrices are computed:

\[
\lambda_m(E) = 0.0146, \quad \lambda_M(E) = 278.365, \quad \text{tr}(E) = 318.984, \quad \bar{\lambda}(E) = 28.9986; \]

\[
\lambda_m(\Delta) = 0.0171, \quad \lambda_M(\Delta) = 325.9, \quad \text{tr}(\Delta) = 373.46, \quad \bar{\lambda}(\Delta) = 33.9509.\]

Consider the performance index (7). It is clear that the following scalar bounds are valid for any initial state \( x_0^T P x_0 = J_i \leq J \leq J_u = x_0^T P_u x_0 \). Let \( x_0 \) be an \( 11 \times 1 \) vector with all entries equal to one. Then, \( J \) represents the sum of all entries of the solution matrix \( P \). The following results are obtained:

\[
144.7214 = J_i \leq J = 221.6909 \leq J_u = 553.3855; \]

\[
x_0^T E x_0 = 349.05, \quad x_0^T \Delta x_0 = 408.6642.\]

It can be verified that the matrix bounds and their parameters provide rather good estimates for the exact solution, which can be characterized by a very high value for the ratio \( \lambda_m(P)/\lambda_M(P) = 22026 \).

Naturally it is expected that the bounding interval is a rather extended one in such cases.

4.2. Interconnected power system

The LE (1) is solved for \( A \), as given in Appendix 2 and \( Q = I \). The scalar parameters in (4) are computed as \( \mu_i = 0.50001537, \mu_u = 0.5007855 \), which supposes rather tight solution estimates. The extremal eigenvalues, traces and average eigenvalues of \( P \) and its lower and upper matrix bounds (4) are given below:

\[
\lambda_m(P) = 0.0050007, \quad \lambda_M(P) = 33.81064, \quad \text{tr}(P) = 77.078, \quad \bar{\lambda}(P) = 9.6347; \]

\[
\lambda_m(P_l) = 0.004998, \quad \lambda_M(P_l) = 33.81063, \quad \text{tr}(P_l) = 77.046, \quad \bar{\lambda}(P_l) = 9.6308; \]

\[
\lambda_m(P_u) = 0.005436, \quad \lambda_M(P_u) = 33.853348, \quad \text{tr}(P_u) = 77.643, \quad \bar{\lambda}(P_u) = 9.7504.\]
The percentage errors in the respective estimates are summarized as it is shown in Table 3.

**Table 3**

<table>
<thead>
<tr>
<th>$\Delta \lambda_n (P_1)$</th>
<th>$\Delta \lambda_m (P_1)$</th>
<th>$\Delta \lambda_m (P_2)$</th>
<th>$\Delta \lambda_m (P_3)$</th>
<th>$\Delta \lambda_m (P_4)$</th>
<th>$\Delta \lambda_m (P_5)$</th>
<th>$\Delta \lambda (P_5)$</th>
<th>$\Delta \lambda (P_6)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>%</td>
<td>%</td>
<td>%</td>
<td>%</td>
<td>%</td>
<td>%</td>
<td>%</td>
<td>%</td>
</tr>
<tr>
<td>0.054</td>
<td>3×10$^{-5}$</td>
<td>0.0732</td>
<td>0.1263</td>
<td>0.04</td>
<td>0.16</td>
<td>0.0405</td>
<td>0.7335</td>
</tr>
</tbody>
</table>

All respective errors are less than 0.13 %, which confirms that all estimates are very tight in this case. This fact is also clear from the computed values for the extremal eigenvalues, traces and average eigenvalues of the average matrix estimate $E = 0.5(P_1 + P_2)$ and the error matrix $\Delta = P_1 - P_2$:

$\lambda_n (E) = 0.005001$, $\lambda_m (E) = 33.832$, tr$(E) = 77.095$, $\lambda (E) = 9.6369$,

$\lambda_n (\Delta) = 0.000006$, $\lambda_m (\Delta) = 0.043$, tr$(\Delta) = 0.0973$, $\lambda (\Delta) = 0.121$.

The performance index and its lower and upper estimates are computed for the same initial state vector as in example 4.1, i.e., $128.99 \leq J \leq 129 \leq J_u = 129.16$, which also illustrates the fact that the obtained solution bounds are rather precise in this case.

**References**


**Appendix 1**

The nominal values of the system’s parameters are as follows:

$t_{G1} = t_{G2} = 0.1 \text{ s}; t_{f1} = t_{f2} = 0.3 \text{ s}; t_{b1} = t_{b2} = 10 \text{ s};$

$t_{p1} = t_{p2} = 20 \text{ s}; k_{p1} = k_{p2} = 120 \text{ Hz/MW}.$
\[ r_1 = r_2 = 2.4 \text{ Hz/MW}; \quad b_1 = b_2 = 0.425 \text{ MW/Hz}; \]

\[ k_1 = k_2 = 0.5; \quad k_{12} = k_{21} = 0.05; \quad T_{12} = 0.0707, a_{12} = 1; \]

\[ a = (k_1 + k_2)/t_{11} - k_1/t_{11} + b = (k_1 + k_2)/t_{12} - k_2/t_{12}. \]

The stabilized close loop system state matrix \( A_s = A - BK \) is partitioned as follows:

\[
A_s = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad A_{11} \in \mathbb{R}_{4}, \quad A_{12} \in \mathbb{R}_{4,6}, \quad A_{21} \in \mathbb{R}_{6,4}, \quad A_{22} \in \mathbb{R}_{6,6};
\]

\[
A_{11} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0.0213 \\ -6.0178 & -16.5623 & -5.7229 & -12.2171 & -3.7546 \\ 0 & 3.3333 & -3.3333 & 0 & 0 \\ 0 & 1.6666 & 1.6667 & -0.1 & 0 \\ -74.0812 & 12.8135 & 12.032 & 33.8268 & -60.9484 \end{bmatrix};
\]

\[
A_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0.4513 & -12.4877 & -14.7628 & -2.6993 & -0.5823 & -3.8753 \\ 0 & 0 & 3.3333 & -3.3333 & 0 & 0 \\ 0 & 0 & 1.6666 & 1.6667 & -10 & 0 \\ 76.2689 & 38.86 & 8.4688 & 4.8353 & 19.2739 & -61.4985 \end{bmatrix};
\]

\[
A_{21} = \begin{bmatrix} 0.05 & 0 & 0 & 0 & 0 & 0 \\ 1.4189 & 0.283 & -0.0186 & -0.028 & -0.0535 & -0.0109 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -72.0432 & 8.5496 & 0.1233 & -0.0173 & -1.1328 & 0.1128 \end{bmatrix};
\]

\[
A_{22} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0.444 \\ 0 & 0 & 0 & 0 & -0.005 \\ 0.1984 & -0.0186 & -0.029 & -0.1529 & 0.0034 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 20.1713 & -0.394 & -0.6452 & -5.9822 & 0.1128 \end{bmatrix}.
\]
Appendix 2

The nominal stable state matrix $A$ of the interconnected power system is

$$
\begin{bmatrix}
-100 & -2.2 & 0 & 0.5 & 0 & 0.3 & 0 & 0.2 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.5 & -100 & -2.6 & 0 & 0.6 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.3 & 0 & 0.6 & -100 & -3 & 0 & 0.1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0.2 & 0 & 0 & 0.1 & -100 & -3.4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
$$